A method to calculate DVCS coefficient functions in the conformal moments space

A. N. Manashov

Max Planck Institute, Munich

FG2926 workshop, Regensburg, 17 February 2023

- **Deeply Virtual Compton Scattering**: $\gamma^* \; N \longrightarrow \gamma \; N'$

Müller 94, Ji 96, Radyushkin 96

$$\mathcal{A}_{\mu\nu}(q,q',p) = i \int d^4x \, e^{-iqx} \langle p' | T\{ j_\mu^{\mathrm{em}}(x) j_\nu^{\mathrm{em}}(0) \} | p \rangle \,.$$

The leading twist approximation

$$\mathcal{A}_{\mu\nu} = -g^{\perp}_{\mu\nu} V_+ + \epsilon^{\perp}_{\mu\nu} V_- + \ldots$$

$$V_\pm(\xi, t, Q^2) = \sum_q e_q^2 \int_{-1}^{1} \frac{dx}{\xi} C_\pm(x/\xi, Q^2/\mu^2)\, F_q^\pm(x, \xi, t, \mu)\,.$$

$C_\pm$ **are the coefficient functions,** $F_q^\pm$ **- the generalized parton distributions (GPD)**

$$\xi = -(\Delta, q)/2(P, q), \qquad t = \Delta^2, \qquad \Delta = p' - p, \qquad P = (p + p')/2,$$

The (axial-) vector nonsinglet coefficient functions

$$C_\pm(x/\xi) = C^0_\pm(x/\xi) + a_s C^{(1)}_\pm(x/\xi) + a_s^2 C^{(2)}_\pm(x/\xi)\ldots, \qquad a_s = \alpha_s/4\pi,$$

LO and NLO CFs

$$C^{(0)}_\pm(x/\xi) = 1/z \mp 1/1-z\,,$$

$$C^{(1)}_\pm(x/\xi) = \frac{C_F}{z}\left(\ln^2 z - (2\pm 1)\,\frac{z}{\bar z}\ln z - 9\right) \mp (z\to\bar z)\,,$$

Ji, Osborne, 98, Belitsky, Müller, 98

$z = \frac{1}{2}(1 - x/\xi)$.

Coefficient functions at NNLO

B. Melic, D. Müller, K. Passek-Kumericki, 2002

V. Braun, A.M., S. Moch, J. Schoenleber, 2020 – 21,

J. Gao, T. Huber, Y. Ji, Y. Wang, 2021,

V. Braun, Y. Ji, J. Schoenleber, 2022

$$C^{(2)}(x) \;=\; \beta_0 C_F\, C^{(2\beta)}(x) + C_F^2 C^{(2P)}(x) + \frac{C_F}{N_c} C^{(2NP)}(x)\,.$$

$$\begin{aligned}
C_\pm^{(P)} = \Big\{ & \frac{2}{z}\Big(6\mathrm{H}_{0000} - \mathrm{H}_{1000} - 2\mathrm{H}_{200} - \mathrm{H}_{1100} - \mathrm{H}_{120} - \mathrm{H}_{210} + \mathrm{H}_{1110}\Big) \mp \frac{2}{\bar z}\,\mathrm{H}_{000} + \frac{2}{\bar z}\,\mathrm{H}_{20} \\
& + \frac{4}{z}\,\mathrm{H}_{110} - \left(\frac{8}{z} - \frac{(2\pm 2)}{\bar z}\right)\mathrm{H}_{100} - \left(\frac{12\pm 1}{\bar z} + \frac{38}{3z}\right)\mathrm{H}_{00} + \left(\frac{3\pm 3}{\bar z} + \frac{28\mp 6}{3z}\right)\mathrm{H}_{10} \\
& + \frac{2}{z}\zeta_2\Big(\mathrm{H}_{11} - \mathrm{H}_2 - \mathrm{H}_{10} - 4\mathrm{H}_{00}\Big) + \frac{2}{\bar z}\left(\frac{218\pm 5}{12} \pm (3\pm 2)\zeta_2 \mp 2\zeta_3\right)\mathrm{H}_0 \\
& + \frac{2}{z}\left(3\zeta_2 + 16\zeta_3 - \frac{32}{9}\right)\mathrm{H}_0 + \frac{1}{z}\Big(\frac{701}{24} + \frac{25\mp 9}{3}\zeta_2 + (41\mp 2)\zeta_3 + 3\zeta_2^2\Big)\Big\} \mp (z \leftrightarrow \bar z),
\end{aligned}$$

$\mathrm{H}_{\vec m}$ – Harmonic polylogarithms

Mellin-Barnes representation for CFFs

Müller, 2006, Müller, Kumerički, Passek-Kumerički 2007

$$V(\xi,t,Q^2) \simeq \int_{c-i\infty}^{c+i\infty} dj\, \xi^{-j-1}\, [i+\tan(\pi j/2)]\, C_j(Q/\mu) F_j(\xi,t,\mu)$$

$$C_j(Q/\mu) = \int_0^1 dz\, z\bar{z}\, C_j^{(3/2)}(1-2z)\, C(z,Q/\mu) = 1 + \alpha_s\, C_j^{(1)} + \alpha_s^2\, C_j^{(2)} + \dots$$

$$C_{\pm}^{(0)}(j) = 1$$

$$C_{\pm}^{(1)}(j) = C_F \times \left(4S_1^2(j) - \frac{4S_1(j)}{j(j+1)} + \frac{2}{j^2(j+1)^2} + \frac{(4\pm 1)}{j(j+1)}\right)$$

$$C_{-}^{(2,\beta)}(j) = -\frac{457}{24} + \frac{62}{9\,j\,(j+1)} - \frac{5}{j^3\,(j+1)^3} - \frac{1}{j^2\,(j+1)^2} + \frac{8S_1}{j^2\,(j+1)^2} - \frac{2\,S_1}{3\,j\,(j+1)} - \frac{4S_1^2}{j\,(j+1)} + \frac{2\,(-1)^j\,S_{-2}}{j\,(j+1)} - \frac{2\,\zeta_2+1}{2\,j\,(j+1)} + \frac{38}{9}S_1 + \frac{8}{3}S_1^3 + \frac{4}{3}S_3 + \frac{20}{3}S_1^2 - \zeta_3 - \frac{14}{3}\,\zeta_2,$$

Methods to calculate $C(j)$

- Expand $C_j^{(3/2)}(z) = \sum_k p_{jk} z^k$, $\int dz z^k H_{\vec{m}}(z) =$ Harmonic Sums $(S_{\vec{m}})$

  Melic, Müller, Passek-Kumerički 2003
- Use recurrence relations for Gegenbauer polynomials

One-loop evolution kernel

$$\mathrm{H} \simeq \widehat{\mathrm{H}} - \mathrm{H}_+$$

$\widehat{\mathrm{H}}, \; \mathrm{H}_+ -$ **sl**$(2,\mathbf{R})$ **invariant operators:**

Eigenvalues $\widehat{\mathrm{H}} \mapsto 2S_1(j)$, $\qquad \mathrm{H}_+ \mapsto 1/j(j+1)$

$$[\mathrm{H}_\omega f](z_1,z_2) = \int_0^1 d\alpha \int_0^{\bar\alpha} d\beta\, \omega(\tau) f(z_{12}^\alpha, z_{21}^\beta) \qquad \tau = \frac{\alpha\beta}{\bar\alpha\bar\beta}, \qquad z_{12}^\alpha = z_1\bar\alpha + \alpha z_2$$

$$\delta_+(\tau) \leftrightarrow \widehat{\mathrm{H}}, \qquad 1 \leftrightarrow \mathrm{H}_+$$

Eigenvalues $$\mathrm{E}_\omega(j) = \int_0^1 d\alpha \int_0^{\bar\alpha} d\beta\, \omega(\tau)\, (1-\alpha-\beta)^{j-1}$$

$$\ln\bar\tau \leftrightarrow -\frac{1}{j^2(j+1)^2}, \qquad \ln\tau \leftrightarrow -\frac{S_1(j)}{j(j+1)}, \qquad \bar\tau \leftrightarrow (-1)^j(2S_{-2}(j)+\zeta_2),$$ etc.

Eigenfunctions:

$$\Psi_j^p(z_1, z_2) \sim \int_0^1 dx e^{-ip(xz_1+\bar{x}z_2)} x\bar{x} G_{j-1}^{(3/2)}(1-2x)$$

1.

$$\int_0^\infty dz_1 \Psi_j^p(z_1, 0) \sim \int_0^1 dx \frac{1}{x} \left(x\bar{x} G_{j-1}^{(3/2)}(1-2x) \right) = 1$$

2.

$$\int_0^\infty dz_1 [\mathrm{H}_\omega \Psi_j^p](z_1, 0) = \mathrm{E}_\omega(j) = \int_0^1 dx f_\omega(x) \left(x\bar{x} G_{j-1}^{(3/2)}(1-2x) \right)$$

$$f_\omega(x) = \int_0^1 d\alpha \int_0^{\bar{\alpha}} d\beta \frac{\omega(\tau)}{x\bar{\alpha} + \beta\bar{x}}$$

If $\quad f_\omega(x) = \int_0^1 d\alpha \int_0^{\bar\alpha} d\beta\, \omega(\tau) f(x\bar\alpha + \beta\bar x) \quad$ then $\quad M_j(f_\omega) = M_j(f)\mathrm{E}_\omega(j)$

$$f(x) = \mathrm{H}_{\vec m}(x) \times \left(1, \frac{1}{x}, \frac{1}{1-x}\right)$$

HyperInt , **E. Panzer, 2014**

We want to calculate Gegenbauer moments of functions $\frac{1}{x}\mathrm{H}_{\vec m}(x)$ up to weight 4,
$\vec m = (\{0\}, \{1\}, \{0,0\}, \ldots, \{0,0,0,0\} \ldots, \{1,1,1,1\})$

- Level 0 one function: $1/x.$; Its Gegenbauer moment $M(j) = 1$
- Level 1 Two HPL functions: $\mathrm{H}_0, \mathrm{H}_1$

  and two invariant kernels $\widehat{H} \mapsto S_1(j), \quad H_+ \mapsto \eta(j) = 1/j(j+1)$.

$$\widehat{H}\frac{1}{x} = \frac{1}{x}\Big(A\, H_0(x) + B H_1(x)\Big),$$
$$H_+\frac{1}{x} = \frac{1}{x}\Big(C\, H_0(x) + D\, H_1(x)\Big)$$

$$\begin{pmatrix} S_1(j) \\ \eta(j) \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} M_0(j) \\ M_1(j) \end{pmatrix}$$

- Level 2: Four HPLs $\mathrm{H}_{00}, \mathrm{H}_{01}, \mathrm{H}_{10}, \mathrm{H}_{11}$. Kernels S_1^2, $\eta\, S_1$, η^2 plus $(-1)^j S_{-2}(j)$

$$\begin{pmatrix} S_1^2(j) \\ \vdots \\ S_{-2}(j) \end{pmatrix} = \begin{pmatrix} A_{11} & \cdots & A_{14} \\ \vdots & \ddots & \vdots \\ A_{41} & \cdots & A_{44} \end{pmatrix} \begin{pmatrix} M_{00}(j) \\ \vdots \\ M_{11}(j) \end{pmatrix} + \begin{pmatrix} F_{00}(j) \\ \vdots \\ F_{11}(j) \end{pmatrix}$$

- Level 3: Eight HPLs $\mathrm{H}_{000}, \ldots, \mathrm{H}_{111}$. Eight kernels $S_1^3, \ldots \eta^3$, $(-1)^j S_{-2}(j) \times (\eta, S_1)$ plus S_3 and $\Omega_{1,-2} = (-1)^j (S_{1,-2} - 1/2 S_{-3})$

- Level 4: Sixteen HPLs. Three new kernels $(-1)^j S_{-4}$, $\Omega_{1,3} = S_{1,3} - \frac{1}{2} S_4$, $(-1)^j \left(\Omega_{1,1,-2} = S_{1,1,-2} - \frac{1}{2} S_{2,-2} - \frac{1}{2} S_{1,-3} + \frac{1}{4} S_4\right)$

Examples

$$C_+^P = 8S_4 - 16S_{1,3} + 32S_3S_1 - 8S_3\,\eta - 8S_{-3} - 4S_{-2}^2 + 16S_{-2,1} - 8S_{-2}S_1\,\eta + 8S_{-2}\eta^2 - 4\zeta_2 S_{-2}$$
$$- 12S_{-2} + 8S_1^4 - 16S_1^3\eta + 28S_1^2\eta^2 + 32S_1^2\eta - 16\zeta_2 S_1^2 - \frac{76}{3}S_1^2 - 32S_1\eta^3 - 92S_1\eta^2$$
$$- \frac{86}{3}S_1\eta + 12\zeta_2 S_1\eta - 12\zeta_2 S_1 - 64\zeta_3 S_1 + \frac{128}{9}S_1 + 10\eta^4 + 50\eta^3 - 2\zeta_2\eta^2 + 49\eta^2$$
$$- 6\zeta_2\eta + 32\zeta_3\eta - \frac{815}{18}\eta - \frac{14}{5}\zeta_2^2 + 43\zeta_3 + \frac{701}{24} + \frac{16}{3}\zeta_2$$

$$C_-^P = 8\,S_4 - 16\,S_{1,3} + 32\,S_3S_1 - 8\,S_3\eta - 4\,S_{-2}^2 + 8\,S_{-2}S_1\eta - 8\,S_{-2}\eta^2 - 4\,\zeta_2 S_{-2} + 8\,S_1^4$$
$$- 16\,S_1^3\eta + 28\,S_1^2\eta^2 + 24\,S_1^2\eta - 16\,\zeta_2 S_1^2 - \frac{76}{3}\,S_1^2 - 32\,S_1\eta^3 - 76\,S_1\eta^2 - \frac{26}{3}S_1\,\eta$$
$$+ 20\,\zeta_2 S_1\eta - 12\,\zeta_2 S_1 - 64\,\zeta_3 S_1 + \frac{128}{9}S_1 + 10\,\eta^4 + 42\,\eta^3 - 10\,\zeta_2\eta^2 + 21\,\eta^2$$
$$- 2\,\zeta_2\eta + 32\,\zeta_3\eta - \frac{1001}{18}\eta - \frac{14}{5}\zeta_2^2 + 39\,\zeta_3 + \frac{701}{24} + \frac{34}{3}\,\zeta_2\,,$$

$\eta = 1/j(j+1).$